Source: Prentice et al. March 1993. A Study to Determine the Biological Feasibility of a New Fish Tagging System, 1989 Annual Report, U.S Department of Energy, Bonneville Power Administration, Division of Fish and Wildlife, and National Oceanic and Atmospheric Administration.

Appendix B: Statistical Method of Determining PIT-Tag Coil Reading Efficiency, pp. 140-144.

By Benjamin Sanford

DISCUSSION

Direct and indirect methods can be used to determine PIT-tag monitor tag-reading efficiency. The direct method compares the number of tagged fish monitored to that of a known number of tagged fish released directly into the monitoring system. This method is only accurate for the time and conditions of the test and does not necessarily represent reading efficiency over a prolonged period. The indirect method is a statistical method based upon the number of tagged fish monitored while not knowing the actual number of fish passing through the system. The following is a description of the derivation of a point estimator, with its associated estimated variance, for the probability of missing a PIT tag with a PIT-tag monitor unit.

Consider a PIT-tag monitor unit consisting of k coils. An unbiased maximum likelihood estimate (MLE) for P_i, the probability of detection on coil i (i = 1,...,k), can be obtained under the following two assumptions:

Al) P_i and P_j are independent for $i \neq j$.

A2) P_i is the same for all PIT tags.

Under A2, we can treat the tags detected on coil i as a random sample of all tags passing through the unit. Incorporating Al as well, we can treat the tags detected on all other coils as a random sample of all tags passing through the unit independent of whether those tags were detected on coil i.

Let $P_{i/j}$ equal the probability of detection on coil i given detection on at least one other coil. Al implies that $P_{i/j} = P_i$

Let n_i equal the number of unique tags detected on coil i and at least one other coil.

Let M_i equal the total number of unique tags detected on at least one other coil.

It is then reasonable to assume that n_i is binomially distributed with parameters M_i and $P_{i/j}$, = P_i.

The unbiased MLE for P_i is then (Mood et al. 1974)

$$p_i = n_i / M_i \qquad (1)$$

The estimated variance of p_i is

$$p_i (1 - p_i) / M_i = n_i (M_i - n_i) / M_i^3 \qquad (2)$$

This method can be repeated for each coil in the unit. Thus, estimates p_i , i = 1,..., k can be obtained for the detection efficiencies of the k coils in a unit. These estimates are independent. Therefore, P_0, the probability of a tag passing a unit undetected, is the product of the probabilities of the tag passing all k coils undetected, i.e., the product of the $(1-P_i)$s. An unbiased estimate for P_0 is then

$$p_0 = \Pi_{i=1}^{k} (1 - p_i) \qquad (3)$$

The in-variance property of MLEs implies that p_0 is the MLE for P_0. The estimated variance of p_0 can be approximated using a Taylor series expansion, i.e., the Delta method (Mood et al. 1974), as follows:

$$\text{var} (p_0) = \text{var} (\Pi_{i=1}^{k} (1 - p_i)) \approx \Sigma_{i=l}^{k} \text{var} (p_i) \{\delta P_0/\delta P_i (p_i,\ldots,p_k) \}^2$$

$$\delta P_0/\delta P_i (p_i,\ldots,p_k) = -\Pi_{j=l}^{k} (1 - p_j) / (1 - p_i) = -p_0 / (1 - p_i)$$

$$\text{Thus, var} (p_0) \approx \Sigma_{i=1}^{k} [p_i (1 - p_i) / M_i] [-p_0 / (1 - p_i)]^2$$

$$= p_0^2 \Sigma_{i=1} p_i / [M_i (1 - p_i)] \qquad (4)$$

$$= p_0^2 \Sigma_{i=1}^{k} n_i / [M_i (M_i - n_i)] \qquad (5)$$

An approximate $(1 - \alpha)$ 100% confidence interval for the probability of missing a tag for a PIT tag monitor unit is:

$$P_0 \pm z_{\alpha/2} p_0 \left(\Sigma_{i=1}^{k} p_i / [M_i (1 - p_i)] \right)^{½} \qquad (6)$$

where α is the desired significance level and $z_{\alpha/2}$ is a standard normal deviate corresponding to $\alpha/2$ (e. g. , α=0.05, $z_{\alpha/2}$= 1.96) .

The estimated probability of missing a tag for an overall monitor system, Π_0 say, is a weighted average of the probabilities for each unit provided the units cover mutually exclusive routes. The estimate, for u units in a system, is

$$\Pi_0 = \Sigma_{i=1}^{u} p_{0i} w_i \qquad (7)$$

where p_{0i} is the estimate, p_0, for unit i, (i = 1,…,u) , and w_i is the weight for unit i.

The estimated variance of Π_0 is

$$\mathrm{var}(\Pi_0) = \Sigma_{i=1}^{u} \mathrm{var}(p_{0i})\, w_i^2 \qquad (8)$$

An approximate $(1 - \alpha)$ 100% confidence interval for the true system probability of missing a tag is

$$\Pi_0 \quad z_{\alpha/2} [\mathrm{var}(\Pi_0)]^{½} \qquad (9)$$

LITERATURE CITED

Mood, A. M., F. A. Graybill, and D. C. Boes. 1974. Introduction to the theory of statistics. McGraw-Hill, Minneapolis, 564 p.