**DISCUSSION**

Direct and indirect methods can be used to determine PIT-tag monitor tag-reading efficiency. The direct method compares the number of tagged fish monitored to that of a known number of tagged fish released directly into the monitoring system. This method is only accurate for the time and conditions of the test and does not necessarily represent reading efficiency over a prolonged period. The indirect method is a statistical method based upon the number of tagged fish monitored while not knowing the actual number of fish passing through the system. The following is a description of the derivation of a point estimator, with its associated estimated variance, for the probability of missing a PIT tag with a PIT-tag monitor unit.

Consider a PIT-tag monitor unit consisting of k coils. An unbiased maximum likelihood estimate (MLE) for $P_i$, the probability of detection on coil $i$ ($i = 1, \ldots, k$), can be obtained under the following two assumptions:

A1) $P_i$ and $P_j$ are independent for $i \neq j$.

A2) $P_i$ is the same for all PIT tags.

Under A2, we can treat the tags detected on coil $i$ as a random sample of all tags passing through the unit. Incorporating A1 as well, we can treat the tags detected on all other coils as a random sample of all tags passing through the unit independent of whether those tags were detected on coil $i$. 


Appendix B: Statistical Method of Determining PIT-Tag Coil Reading Efficiency, pp. 140-144.

By Benjamin Sanford
Let $P_{ij}$ equal the probability of detection on coil $i$ given detection on at least one other coil.  All implies that $P_{ij} = P_i$

Let $n_i$ equal the number of unique tags detected on coil $i$ and at least one other coil.

Let $M_i$ equal the total number of unique tags detected on at least one other coil.

It is then reasonable to assume that $n_i$ is binomially distributed with parameters $M_i$ and $P_{ij} = P_i$.

The unbiased MLE for $P_i$ is then (Mood et al. 1974)

$$p_i = \frac{n_i}{M_i} \quad (1)$$

The estimated variance of $p_i$ is

$$p_i (1 - p_i) / M_i = n_i (M_i - n_i) / M_i^3 \quad (2)$$

This method can be repeated for each coil in the unit. Thus, estimates $p_i$, $i = 1, ..., k$ can be obtained for the detection efficiencies of the $k$ coils in a unit. These estimates are independent. Therefore, $P_0$, the probability of a tag passing a unit undetected, is the product of the probabilities of the tag passing all $k$ coils undetected, i.e., the product of the $(1 - P_i)$s. An unbiased estimate for $P_0$ is then

$$p_0 = \prod_{i=1}^{k} (1 - p_i) \quad (3)$$

The in-variance property of MLEs implies that $p_0$ is the MLE for $P_0$. The estimated variance of $p_0$ can be approximated using a Taylor series expansion, i.e., the Delta method (Mood et al. 1974), as follows:

$$\text{var} (p_0) = \text{var} (\prod_{i=1}^{k} (1 - p_i)) = \sum_{i=1}^{k} \text{var} (p_i) \left\{ \delta P_0 / \delta P_i (p_i, ..., p_k) \right\}^2$$

$$\delta P_0 / \delta P_i (p_i, ..., p_k) = -\prod_{j=1}^{k} (1 - p_j) / (1 - p_i) = -p_0 / (1 - p_i)$$

Thus, $\text{var} (p_0) = \sum_{i=1}^{k} [p_i (1 - p_i) / M_i] [-p_0 / (1 - p_i)]^2$

$$= p_0^2 \sum_{i=1}^{k} p_i / [M_i (1 - p_i)] \quad (4)$$
\[ = p_0^2 \sum_{i=1}^{k} n_i / \left[ M_i ( M_i - n_i ) \right] \quad (5) \]

An approximate (1 - \( \alpha \)) 100% confidence interval for the probability of missing a tag for a PIT tag monitor unit is:

\[ P_0 \pm z_{\alpha/2} p_0 \left( \sum_{i=1}^{k} p_i / \left[ M_i ( 1 - p_i ) \right] \right)^{1/2} \quad (6) \]

where \( \alpha \) is the desired significance level and \( z_{\alpha/2} \) is a standard normal deviate corresponding to \( \alpha/2 \) (e. g., \( \alpha = 0.05 \), \( z_{0.025} = 1.96 \)).

The estimated probability of missing a tag for an overall monitor system, \( \Pi_0 \) say, is a weighted average of the probabilities for each unit provided the units cover mutually exclusive routes. The estimate, for \( u \) units in a system, is

\[ \Pi_0 = \sum_{i=1}^{u} p_{0i} w_i \quad (7) \]

where \( p_{0i} \) is the estimate, \( p_0 \), for unit \( i \) (\( i = 1, \ldots, u \) ), and \( w_i \) is the weight for unit \( i \).

The estimated variance of \( \Pi_0 \) is

\[ \text{var} ( \Pi_0 ) = \sum_{i=1}^{u} \text{var} ( p_{0i} ) w_i^2 \quad (8) \]

An approximate (1 - \( \alpha \)) 100% confidence interval for the true system probability of missing a tag is

\[ \Pi_0 \pm z_{\alpha/2} \left[ \text{var} ( \Pi_0 ) \right]^{1/2} \quad (9) \]

LITERATURE CITED